Equation of state and radii of finite nuclei in the presence of a diffuse surface layer

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We have applied [1] the approach proposed earlier by Gibbs-Tolman-Widom for a classical liquid drop in presence of the liquid-vapor interface to the derivation of actual size of a nucleus in presence of finite surface diffuse layer. The basic idea of the Gibbs-Tolman-Widom approach is the introduction of a sharp dividing surface, $\mathcal{S}$. The dividing surface is arbitrary but located within the surface diffuse layer. The actual (physical) equimolar surface and thereby the actual nuclear surface are fixed by the requirement that the contribution to the surface energy $E_S[R] \sim A^{2/3}$ from the bulk energy $E_{\text{bulk}} \sim A$ should be eliminated. The bulk density, ρ_0, of neutrons and protons inside the sharp equimolar surface is obtained using the experimental data for the separation energy s_q for each kind of nucleon.

The Gibbs-Tolman-Widom conception of sharp equimolar surface allows one to derive the nuclear volume and, as a consequence, the pressure $P(\rho_0)$ and the equation of state for finite nuclei. In our consideration, we have performed the calculations of well-defined equation of state for spherical nuclei and some nuclear characteristics such as the nuclear radius, the surface tension, the pressure, etc. Our numerical calculations are based on the direct variational method, the extended Thomas-Fermi approximation and the effective Skyrme nucleon-nucleon interaction. Applying the Gibbs-Tolman-Widom approach, we redefine the surface and symmetry energies. Note that we do not use the traditional leptodermous approximation and evaluate the Coulomb energy taking into consideration the finite diffuse layer of the proton density distribution.

Performing the analysis of the equation of state $P = P(\rho_0)$, we have extracted from $P(\rho_0)$ the partial contributions which occur due to the different sources: the A- and $X = (N - Z)/A$-independent bulk pressure $P_{\text{vol}}(\rho_0)$ caused by the bulk energy of a symmetric nuclear matter; the surface (capillary) pressure, $P_{A,\text{capil}}(\rho_0, X) \sim A^{-1/3}$; the contribution from the symmetry energy, $P_{A,\text{sym}}(\rho_0, X) \sim X^2$ and the Coulomb force contribution $P_{A,\text{C}}(\rho_0, X)$. The corresponding numerical results are given in Ref. [1]. The inclusion of surface (capillary) term $P_{\text{capil}}(\rho_0, X)$ shifts the equilibrium point $\rho_{0,\text{eq}}$ to larger values with respect to the ones in a nuclear matter. Also the capillary pressure $P_{A,\text{capil}}(\rho_0, X)$ is connected to the surface tension coefficient $\sigma(A, X)$ by the classical Laplace relation. The action of the Coulomb pressure $P_{A,\text{C}}(\rho_0, X)$ is opposite to the capillary pressure $P_{A,\text{capil}}(\rho_0, X)$ and shifts the equilibrium point to the smaller values of $\rho_{0,\text{eq}}$.

The use of the Gibbs-Tolman-Widom equimolar radius R_e allowed us to give a more realistic procedure for an extraction of the nuclear surface tension coefficient from the experimental data. The equimolar radius R_e determines the equimolar surface area S_e in absence of a diffuse layer. This fact allows us to evaluate both the surface energy, E_{S_e}, and the surface tension coefficient, $(A, X) = E_{S_e}/S_e$. Using the experimental data within the wide interval of mass number $40 \leq A \leq 220$ and the corresponding values of equimolar radii, we have established the following A-expansion for the surface tension coefficient $\sigma(A, X^*) = \sigma_0 + \sigma_1 A^{-1/3}$ with $\sigma_0 = (0.98 \pm 0.03)\ \mathrm{MeV\,fm^{-2}}$ and $\sigma_1 = (0.75 \pm 0.16)\ \mathrm{MeV\,fm^{-2}}$. The obtained result for the curvature correction $\sigma_1 A^{-1/3}$ allows one to estimate the Tolman length, ξ, in nuclei which is $\xi = (-0.41 \pm 0.07)$ fm.

We have evaluated the partial pressure $P_{A,\mathrm{sym}}(\rho_0, X)$ caused by the symmetry energy. The partial pressure $P_{A,\mathrm{sym}}(\rho_0, X)$ induces the polarization effect on the particle density $\rho_{0,X}$ beyond beta-stability line. We have shown that the partial pressure $P_{A,\mathrm{sym}}(\rho_{0,\mathrm{eq}})$ is positive and reduces the particle density $\rho_{0,X}$ with respect the corresponding equilibrium density $\rho_{0,\mathrm{eq}}$ on the beta-stability line. The partial pressure $P_{A,\mathrm{sym}}$ and the polarization effect are rather sensitive to the Skyrme interaction parameterization such as the SkM*, SLy230b and KDE0v1 interactions. We point out that the evaluated equimolar radius R_e of the nuclei does not necessary obey the saturation condition $R_e = r_0 A^{1/3}$. That is caused by the fact that we use the experimental data for the chemical potentials to derive the bulk density within the equimolar surface in agreement with the Gibbs-Tolman-Widom method. The corresponding experimental chemical potentials (separation energy of nucleons) include the quantum shell effects, the pairing correlation effects, etc., and give rise to the non-monotonic behavior of the nuclear equimolar radii $R_e(A)$. Also that the average interparticle distance r_0 becomes slightly A-dependent.

Using the partial equimolar radii $R_{e,q}(A)$ separately for both kind of nucleons, we have evaluated the corresponding nucleon rms radii $\sqrt{\langle r_q^2 \rangle}$ and the neutron-skin thickness $\Delta r_{np} = \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle}$. The evaluated values of the proton rms radius $\sqrt{\langle r_p^2 \rangle}$ for the Na isotopes show a slightly non-monotonic behavior of $\sqrt{\langle r_p^2 \rangle}$ which is caused by the above mentioned fluctuations of $R_{e,p}$. Also the presence of the significant shift up of the proton rms $\sqrt{\langle r_p^2 \rangle}$ caused by the surface layer corrections. The influence of the pairing and shell effects on the neutron-skin thickness Δr_{np} is illustrated in Fig. 1 for the Na isotopes. As seen from Fig. 1 the Gibbs-Tolman-Widom concept of the sharp equimolar surface allows one to describe a fine non-monotonic structure of the isovector shift Δr_{np}. The saw-like behavior of Δr_{np} (see the open circles connected by the dotted line in these figures) reflects the even-odd and shell effects in the nuclear binding energy and thereby in the nuclear radii. In general, the value of the isovector shift Δr_{np} is the sum of two contributions: the one, $\Delta r_{np,R}$, is due to the different radii (skin effect) and the other, $\Delta r_{np,a}$, is due to the different shape (surface layer) of neutron and proton distributions (halo effect) [1]. One can expect that the neutron halo effect appears more significantly in light nuclei far away from the stability line.

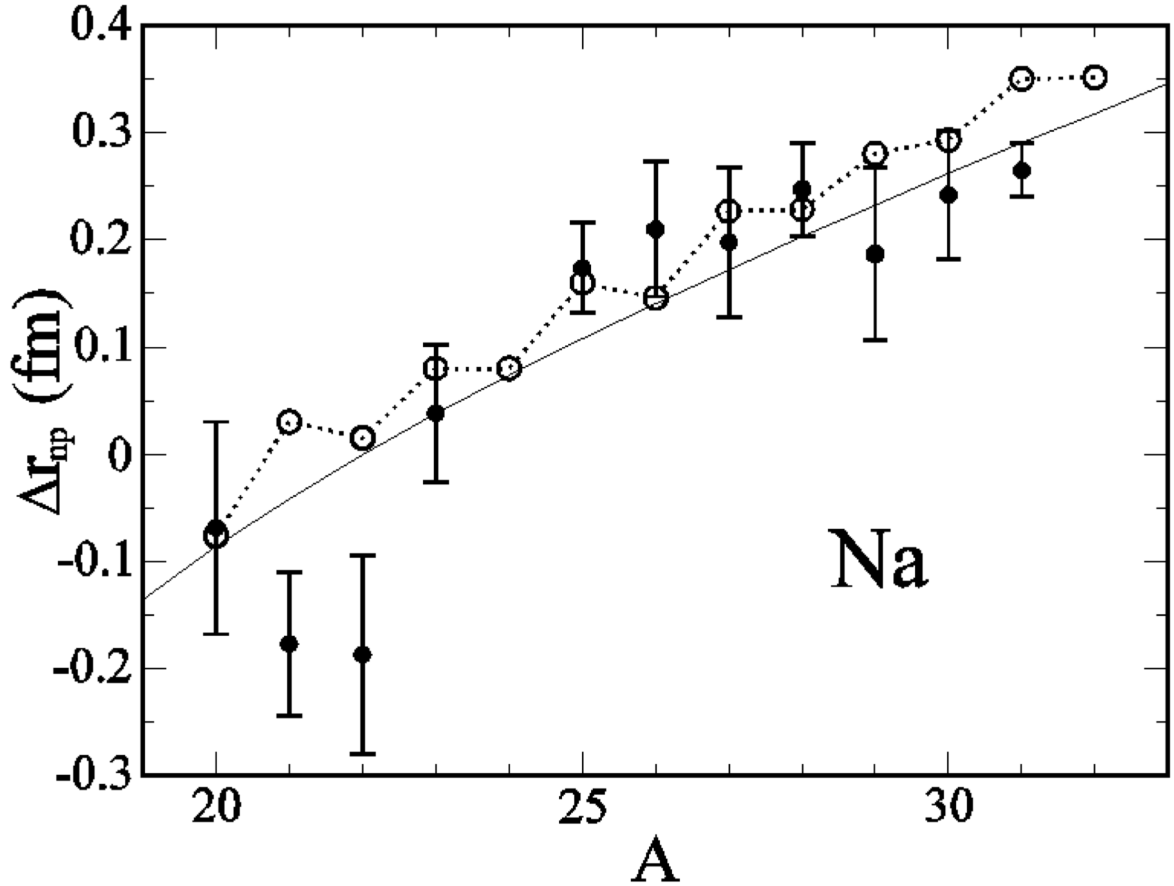

FIG. 1. Isovector shift of nuclear rms radius $\boldsymbol{\Delta r_{np}} = \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle}$ in Na isotopes. The solid points are the experimental data, the open circles (connected by dotted line) have been obtained using the Gibbs-Tolman approach described in text and the solid line is obtained using the extended Thomas-Fermi approximation with the $\mathbf{SkM}^*$ Skyrme interaction.

[1] V.M. Kolomietz, S.V. Lukyanov, A.I. Sanzhur, and S. Shlomo, Phys. Rev. C **95**, 054306 (2017).